1a.) We have the following piecewise polynomials and conditions given for the quadratic spline.

Conditions:

1. 2. 3. 4.

First, we must find , , and . This can be done with the following equations:

Now we must find , , and for . We get the following:

Using these equations, we can get , , and for .

1b.A screenshot of a computer program

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2a.) The order of accuracy for this quadratic spline should be three. This is because the spline uses quadratic polynomials where the error is contained in the cubic term with . α is consistent with this expectation.

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2b.) The splines would be ranked as follows:

1. Clamped cubic spline – 4th order of accuracy

* Advantages: Smallest error
* Disadvantages: Needs at both endpoints

1. Not-a-knot spline – 4th order of accuracy

* Advantages: 4th order of accuracy, does not need endpoint conditions
* Disadvantages: Third derivatives must be continuous

1. Quadratic spline – 3rd order of accuracy

* Advantages: Does not need to compute a column
* Disadvantages: Needs at one endpoint

1. Natural cubic spline – 2nd order of accuracy

* Advantages: Simple endpoint conditions
* Disadvantages: Low order of accuracy

Natural cubic spline does not work well for this problem since the boundary conditions are set so that . Since , , and , the boundary conditions of the natural cubic spline inhibit its performance interpolating .

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3a.) quadspline2 will maintain the order of accuracy of quadspline since was derived using the five-point forward difference formula. Since the order of accuracy for the five-point forward difference formula is greater than that of quadspline, quadspline will maintain its order of accuracy.

3b.)

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3c.) Yes, the convergence order is the same.

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3d.) Aside from different finite difference approximations of , I cannot think of any other ways to compute the quadratic spline without . Since we need , and comes from the derivative of , we need the derivative of for the quadratic spline.